CS 309: Autonomous Robots
FRI I

Coordinate Frames &
Spatial Transformations

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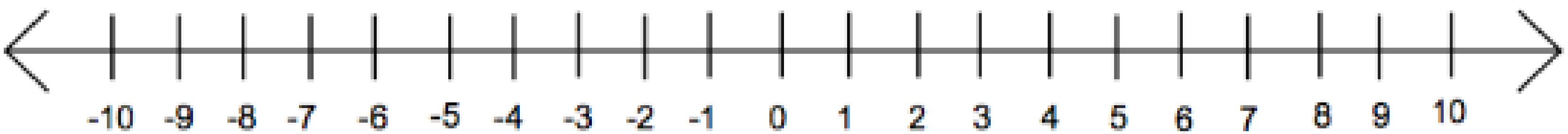
Coordinate Frames

- We use coordinate frames to discuss the pose of the robot and objects in the environment
  - Position – Where something is, translation
  - Orientation – The way it is turned, rotation
  - Pose – Position & Orientation

- Coordinate frames describe
  - Robot on the map
  - Joints in a robot's arm
  - Everything that can be seen or tracked.

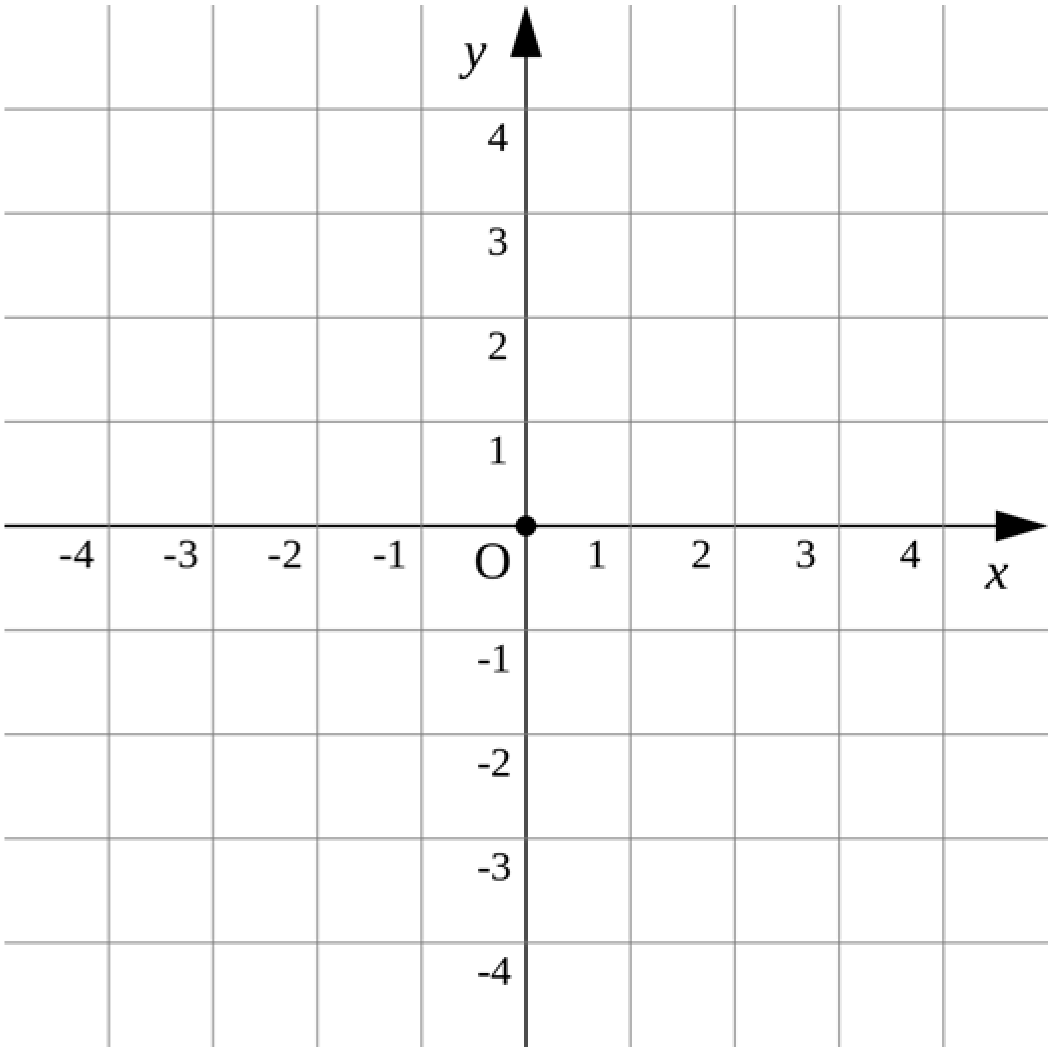
The number line

When you first discussed numbers as a concept
(rather than a quantity)
you probably used a number line



The XY plane

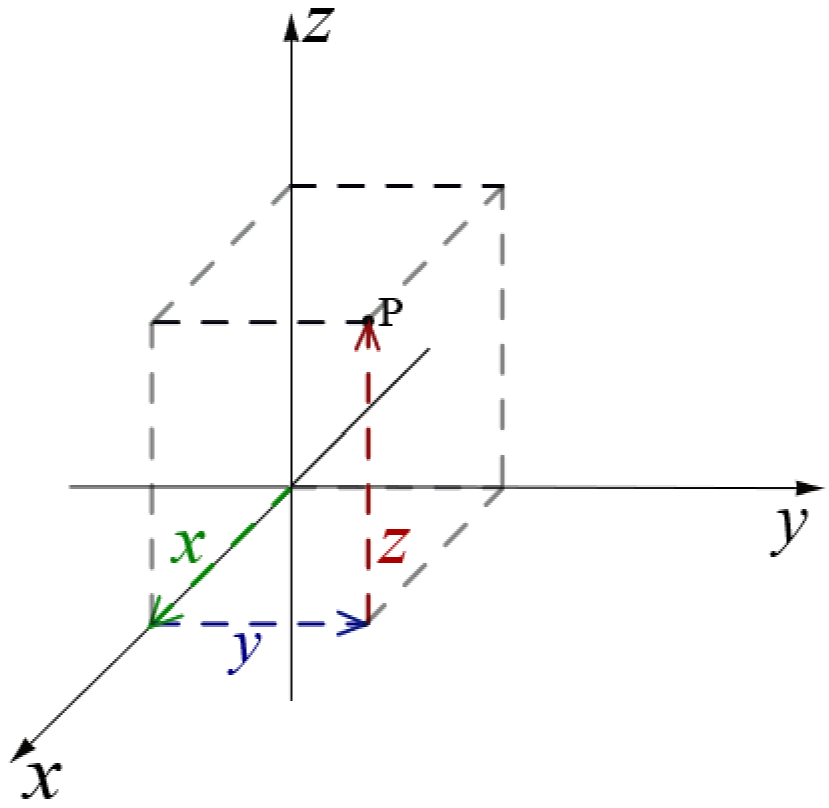
The XY plane you almost certainly used in high school algebra is conceptually similar, but in two dimensions.



XYZ coordinates

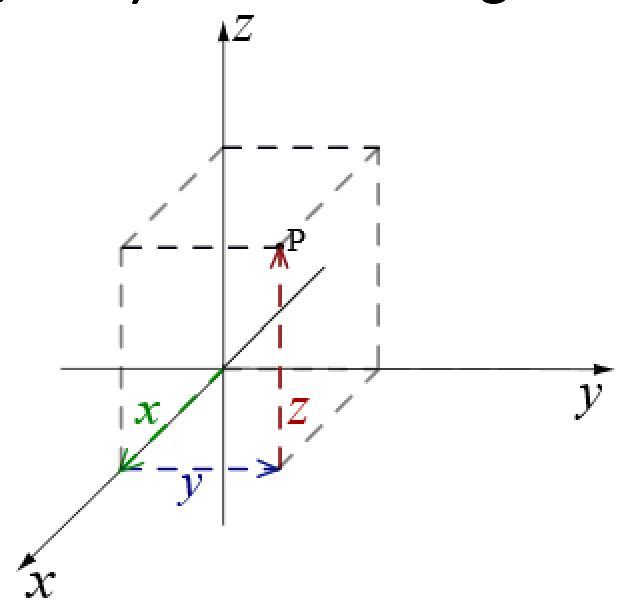
This can be further generalized to 3D

- We typically do use linear algebra terms:
  - Basis, Coordinate Frame, Axes, Transformation, Origin, Vector



- Origin
  - The position at the center of your measurements
    - Think about it, you can only measure a position with respect to some fixed point in space

- Vector
  - Vectors have
    - Direction – The direction the vector is pointing
    - Magnitude – How far it points in that direction
  - To simplify, think of it as an arrow pointing away from the origin
  - Written $\vec{p}$ = <x,y,z>

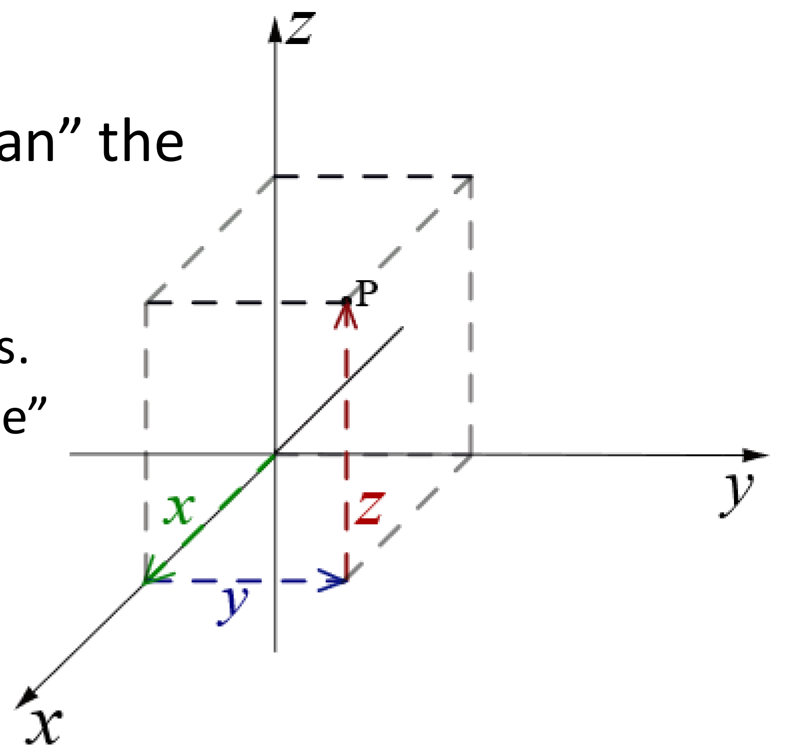


- Axes, Basis, Coordinate Frame
  - There are x,y, & z axes in this graphic
  - They are perpendicular, and point away from the origin
  - They are used to measure your other vectors in space

    $\vec{x}$ = <1,0,0>

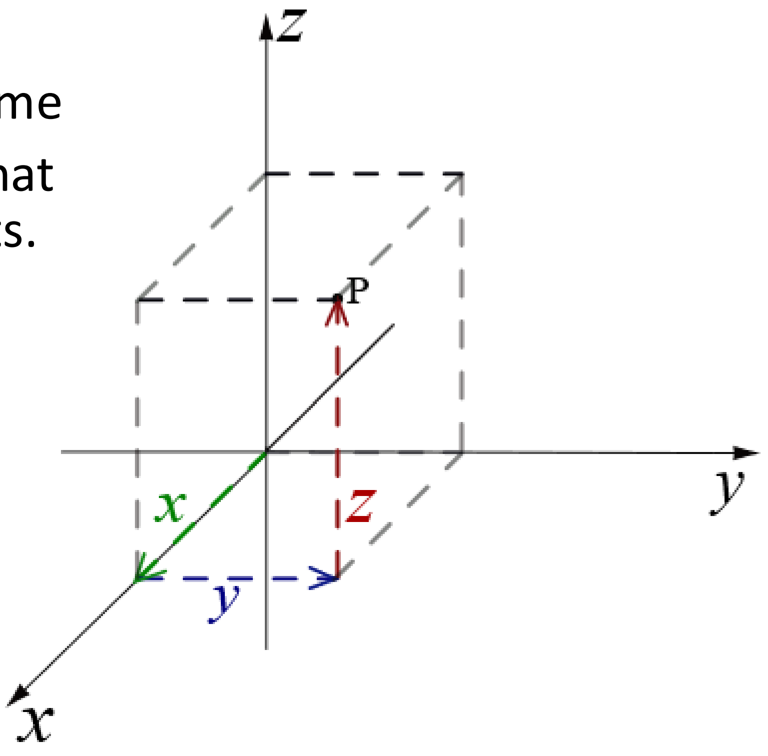
    $\vec{y}$ = <0,1,0>

    $\vec{z}$ = <0,0,1>
  - So, $\vec{p}$ can be stated in terms of how many x, y, and z are needed to arrive at its position
    - $\vec{p}$ = <3,2,4.3>
  - A basis is the set of vectors that "span" the space you want to measure
    - Think of this as having enough vectors to measure the space in all dimensions.
    - We will use the term "coordinate frame" interchangeably.



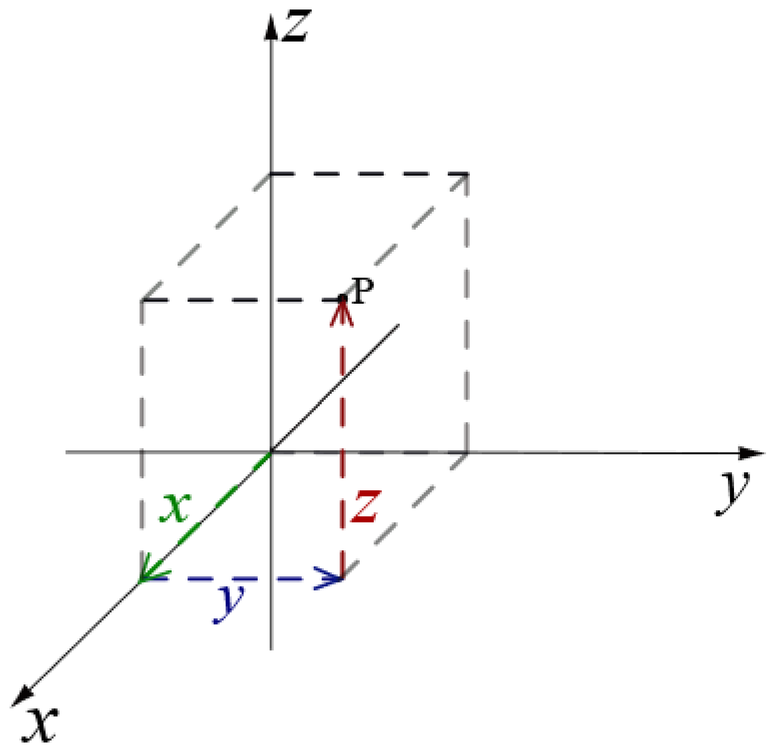
- Transformation
  - Changes measurements in one basis into measurements in another basis

- Common transformations
  - Rotation
  - Translation
  - Rigid Transformation
    - Rotating and translating at the same time
    - More accurately, any transformation that preserves the distances between points.



Let’s revisit this image

- The intersection of x, y, & z is the origin
- x, y, & z are basis vectors, or axes
- The point P’s coordinates are
  - How much is it like x?
  - How much is it like y?
  - How much is it like z?



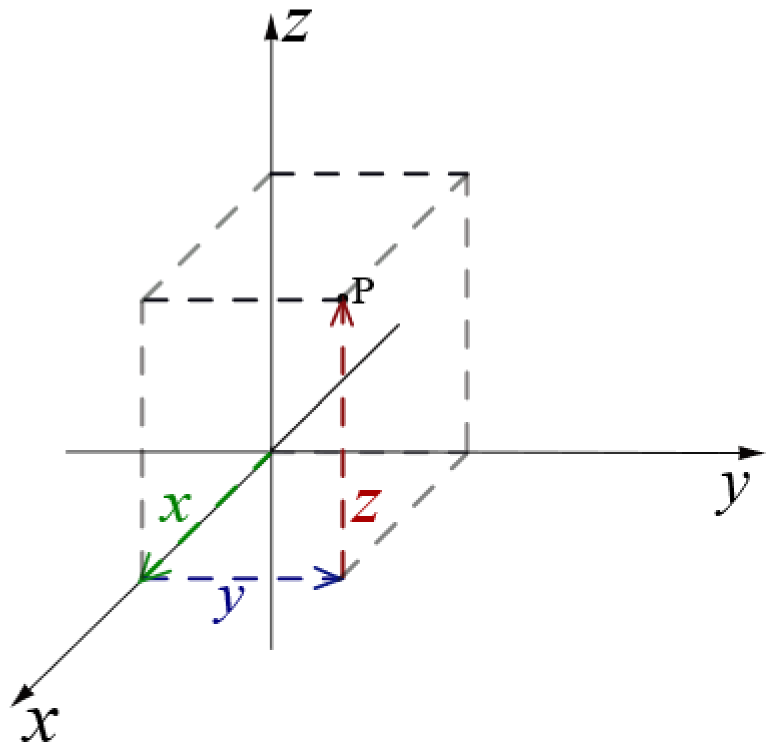
- Vectors

  $\vec{x}$ = <1,0,0>

  $\vec{y}$ = <0,1,0>

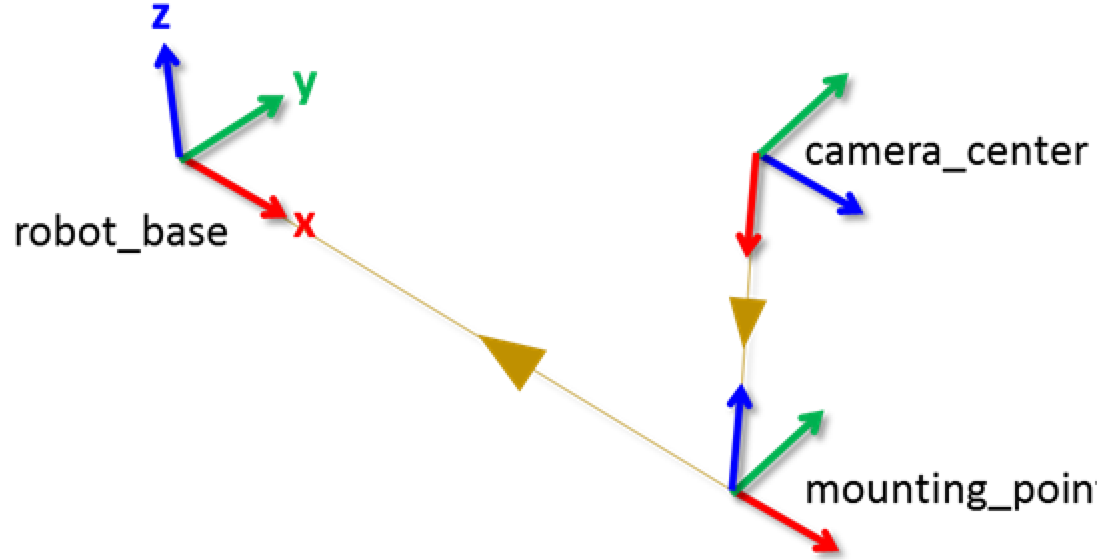
  $\vec{z}$ = <0,0,1>

- If $\vec{p}$ is at <0,0,2>
  - It is 0 like x
  - It is 0 like y
  - It is 2 like z



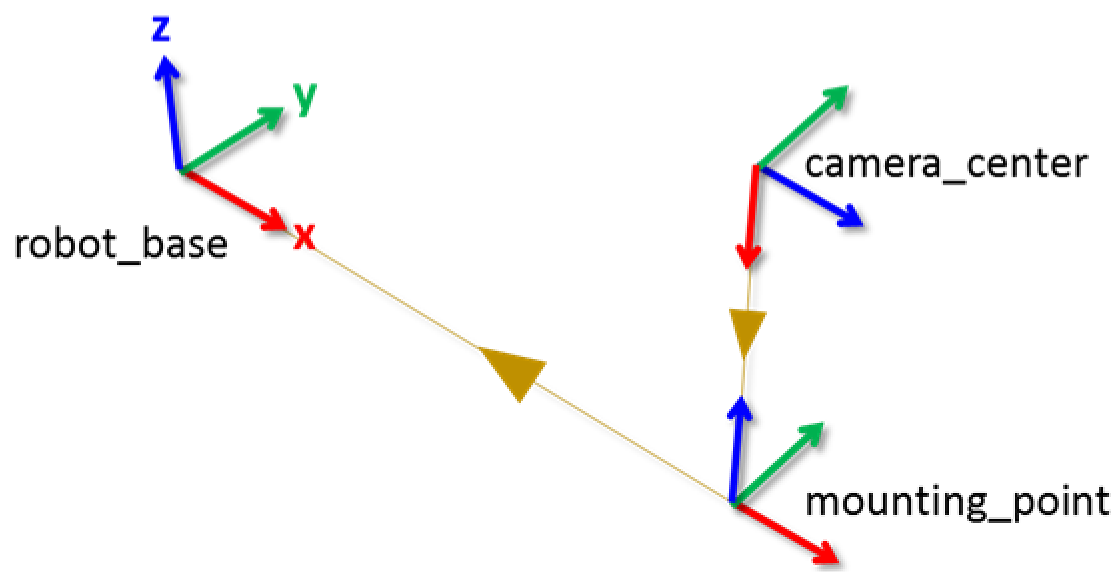
Coordinate frames on our robot

- There are multiple coordinate frames that describe our robot
- Here is a description of where the camera is on the BWIBot
  - robot_base – Coordinate frame of the wheeled base that moves the robot
  - mounting_point – Where the camera is physically mounted
  - camera_center – Based on the camera's lens. Where pictures are taken from.



Transformations

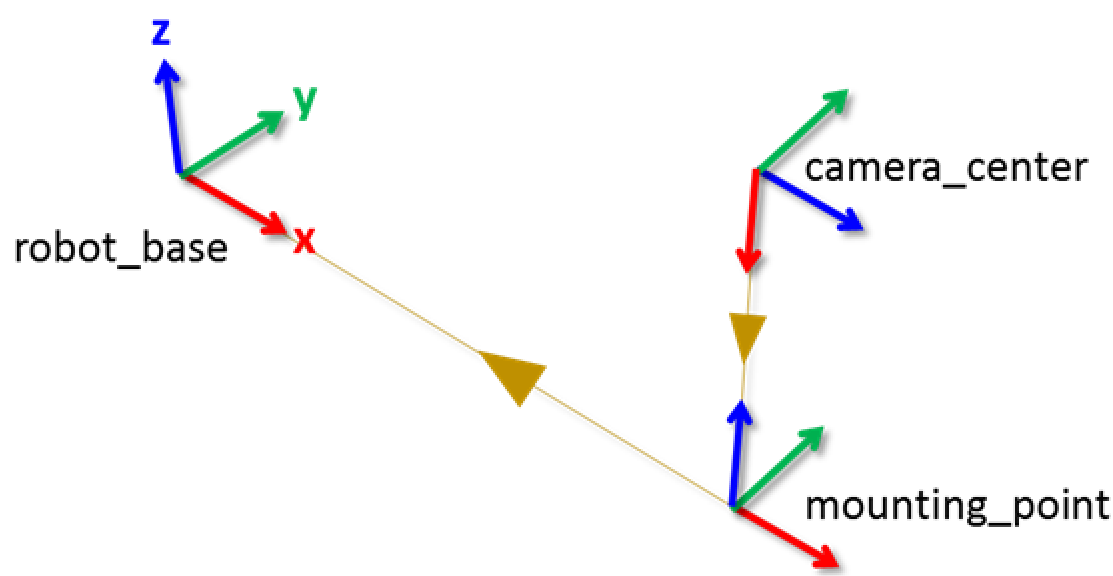
- We think of coordinate frames as relative to each other
- A transformation transforms points from one basis into another basis, or, equivalently, represents the relationship between two coordinate frames.
- For this class we will discuss:
  - Rotation
  - Translation
  - Rigid Transformations
- More advanced:
  - Projections
  - Other transformations



Transformations

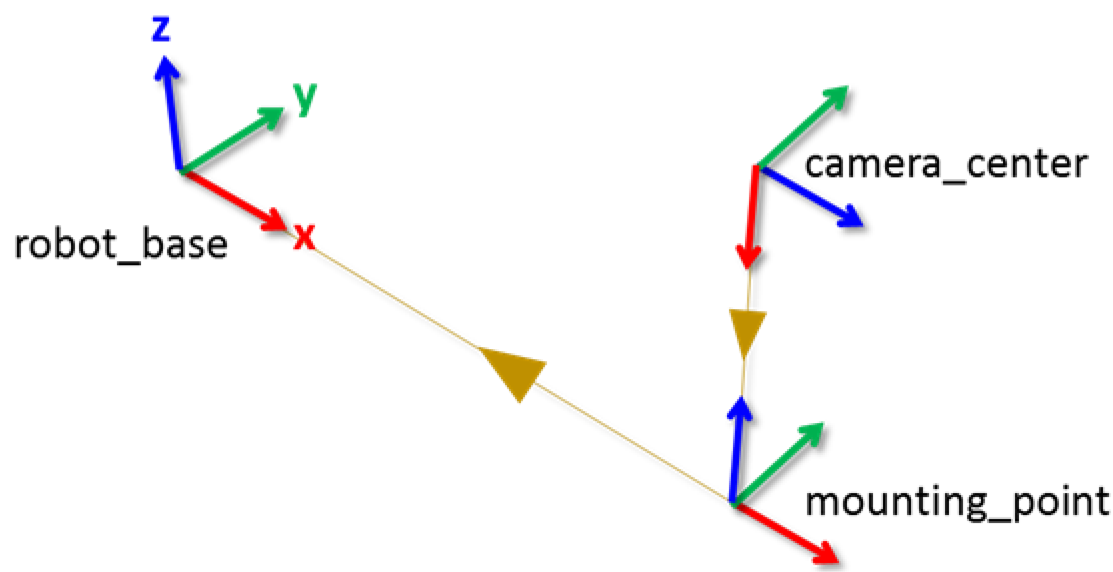
- Each of these coordinate frames
  - Has an origin at <0,0,0>
  - Has its origination such that the vectors face

    <1,0,0>
    <0,1,0>
    <0,0,1>
    ..relative to itself



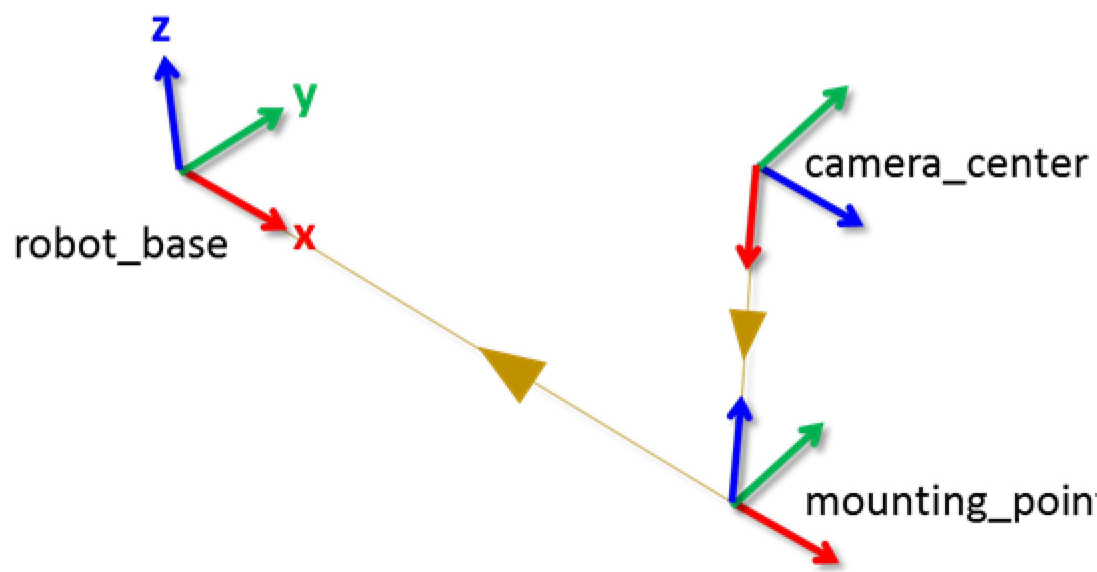
Transformations

- Rotating and translating points expressed relative to robot_base can make them relative to camera_center
- To do this
  - Rotate the points by the rotation that makes the robot_base axes parallel to their equivalent axes in camera_center
  - Translate the points by the translation that would put the robot_base origin on top of the camera_center origin



Rigid transformations

- Rigid transformations combine rotation and translation
- It is a single matrix multiplication that rotates and then translates
  - Internally ROS does this in the TF service, which we will use in this class.



Coordinate frames on a robot arm

- Think of a moving robot arm
- Each joint rotates
- Each joint has a static offset from other joints. The “linkage.”
- Each joint sits on its own coordinate frame
- So tracking coordinate frames tells you where each joint is. It especially tells you where the “end effector” – the robot’s hand – is.