CS 309: Autonomous Intelligent Robotics
FRI I

Lecture 15:
Coordinate Frames & Spatial Transformations

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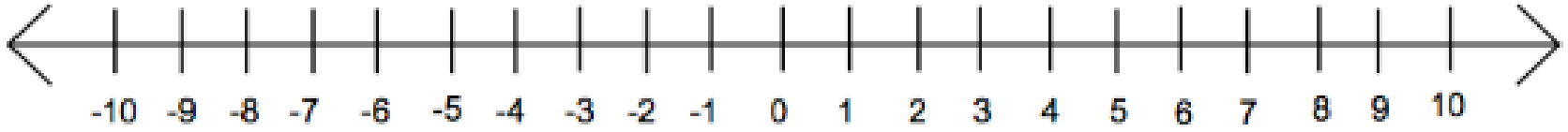
Coordinate Frames

- We’re interested in coordinate frames so we can discuss the position of the robot and its parts.
- Coordinate frames pose the robot on the map, the joints of the robot’s arm, things that the robot sees, and everything else whose motion or position can be tracked.
- We’re going to build from the basics, as this can get quite advanced quickly.

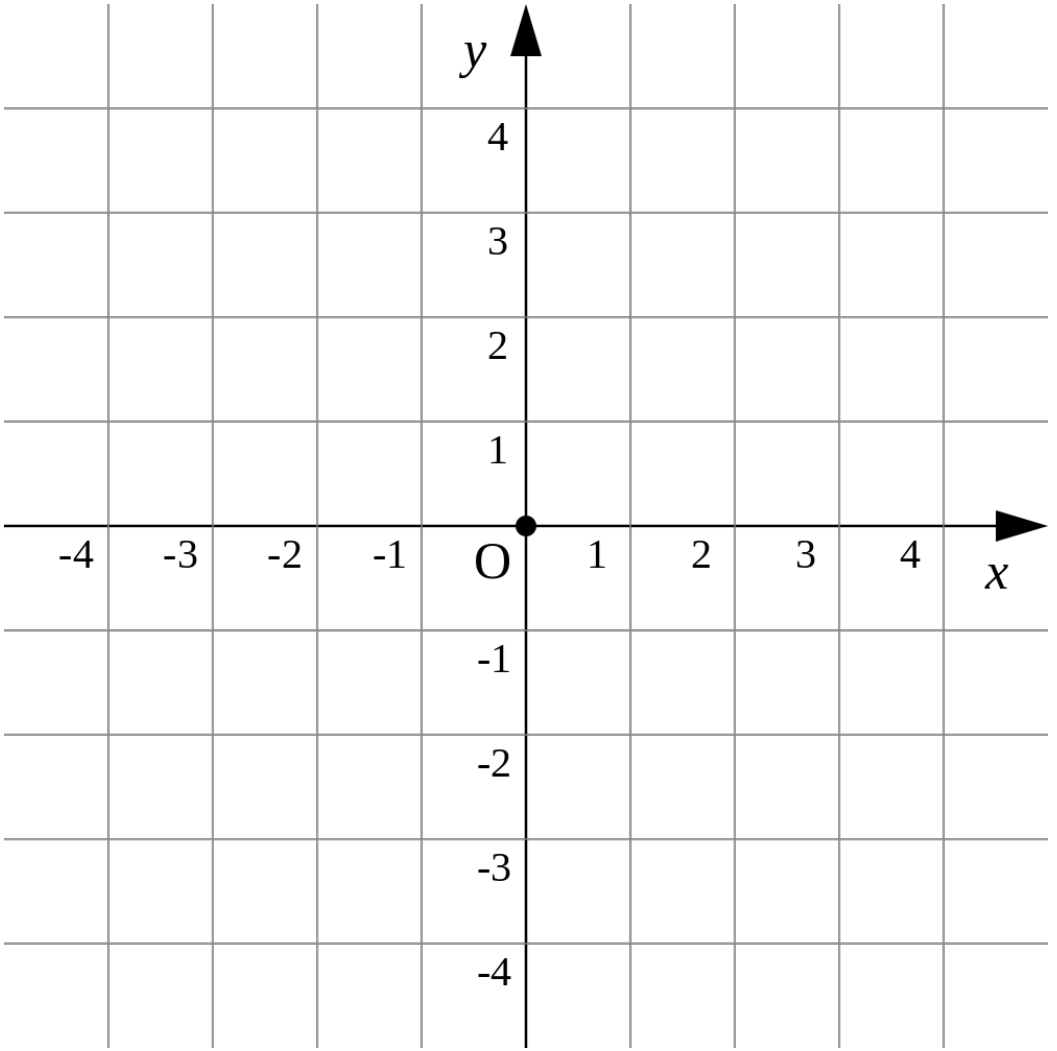
The number line

- When you first discussed numbers as a concept in algebra or arithmetic.



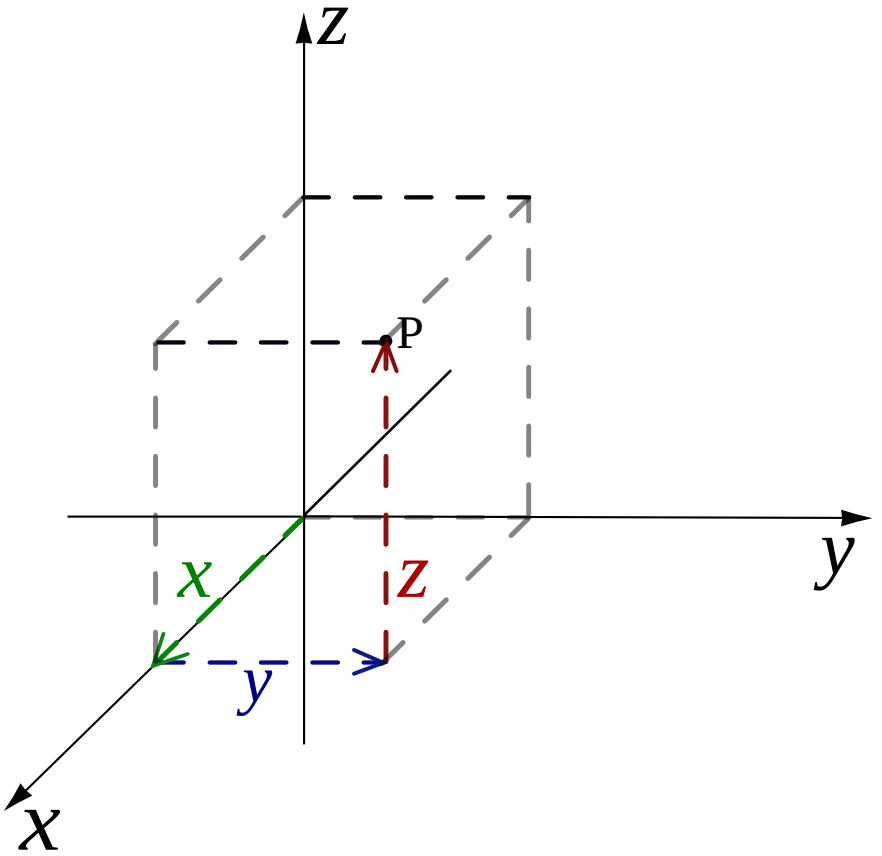
XY Plane

- Next, you discussed the XY Plane



XYZ Coordinates

- We can further generalize this to 3D



The mathematics of this

- Generally, this process is done using linear algebra, and the language used to describe it reflects this.

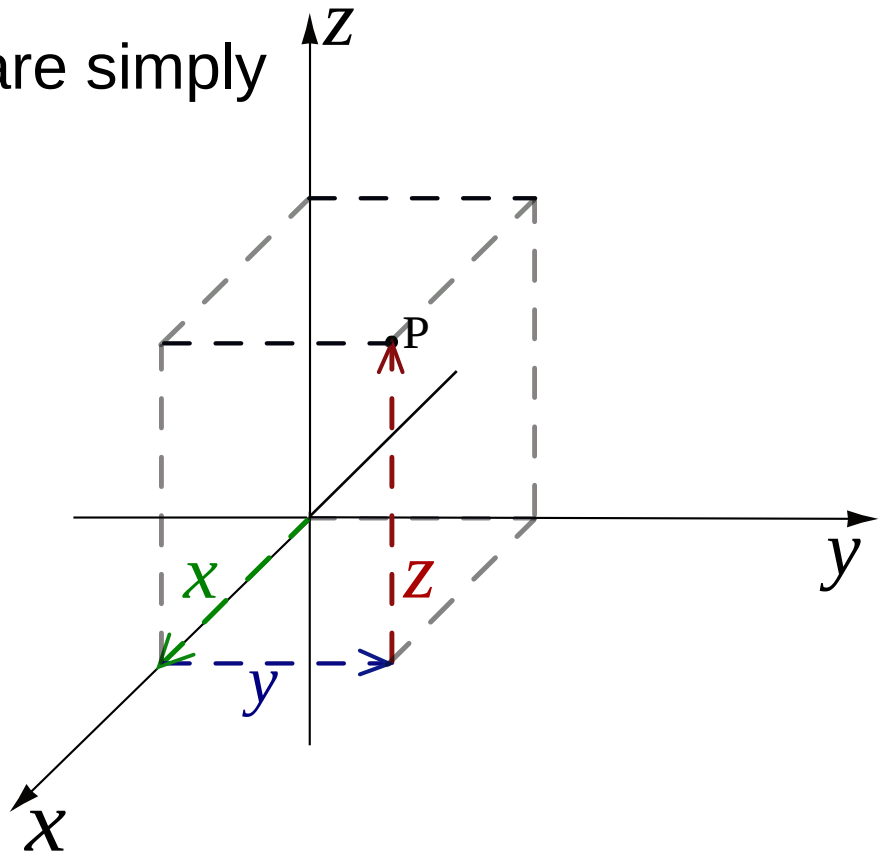
- Origin
  - The position at the center of your measurements
- Vector
  - Vectors have direction and magnitude
    - Think of it as an arrow pointing from the origin in some direction

The mathematics of this

- Basis
  - A set of vectors “spanning” the space you measure
  - These vectors are what you measure things against
- Transformation
  - Changes measurements from one basis to another basis
  - We will simplify this concept in the next few slides.

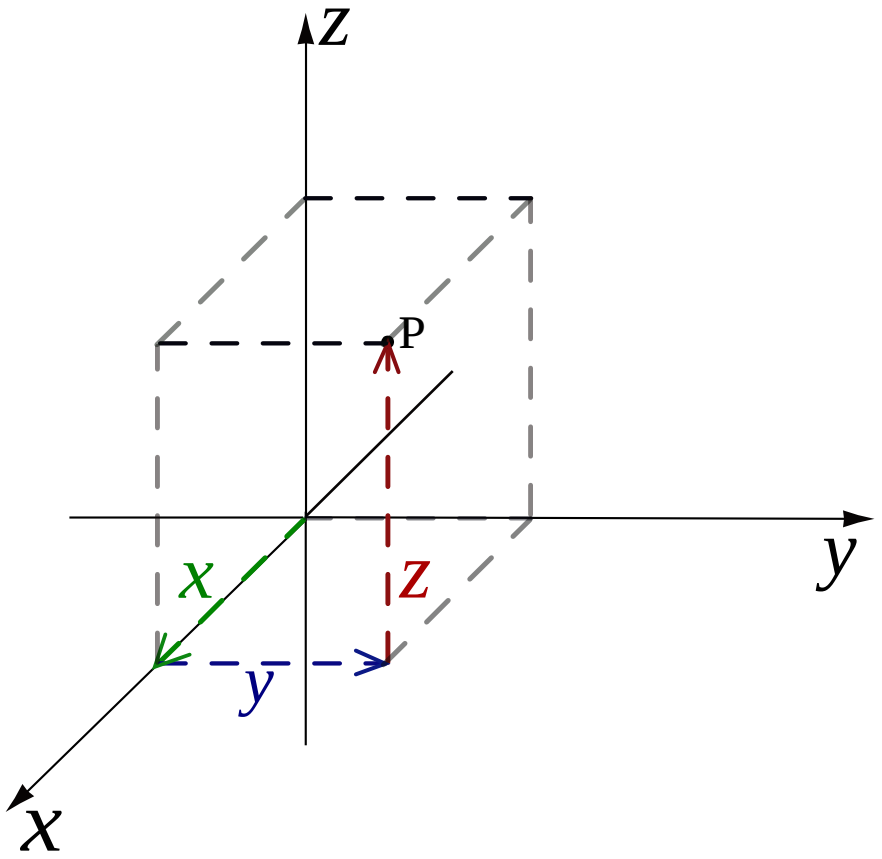
Coordinate Frames

- Revisit this previous image
  - The center is the origin
  - X, Y, and Z are vectors
    - So the point P's coordinates are simply
      - How much is it like X
      - How much is it like Y
      - How much is it like Z



Coordinate Frames

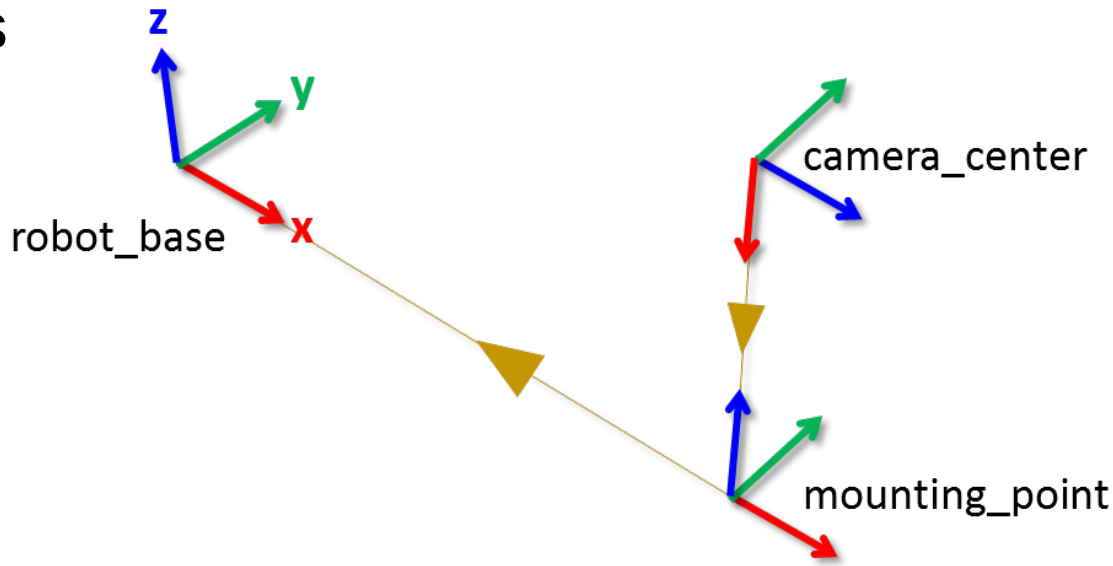
- Vectors
  - X: <1,0,0>
  - Y: <0,1,0>
  - Z: <0,0,1>
- If a point is at (0,0,2)
  - It is 0 like X
  - 0 like Y
  - And 2 like Z



Coordinate Frames

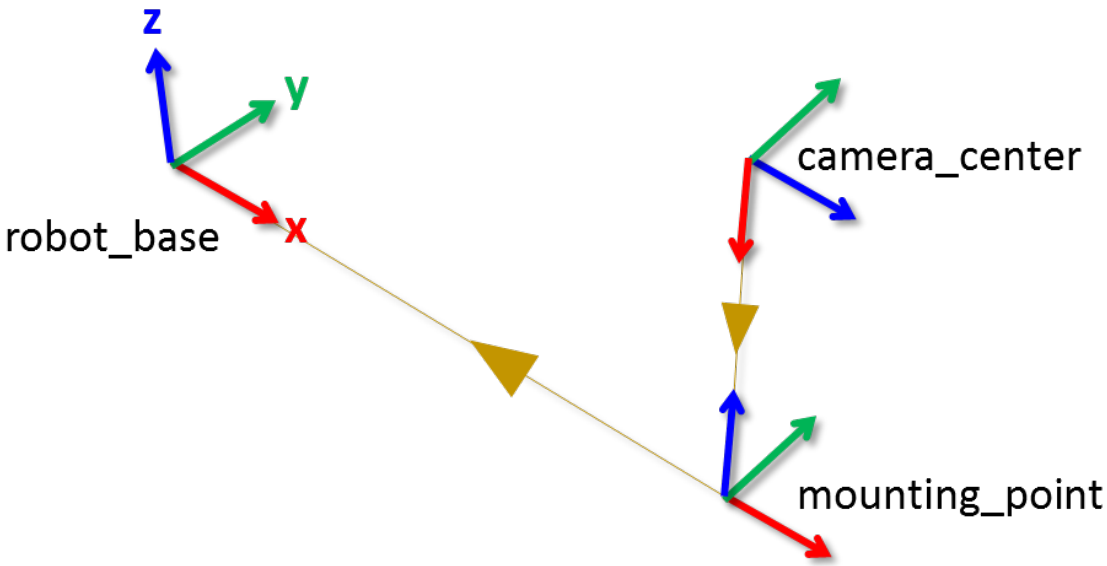
- With our robot, there will be multiple coordinate frames.
  - Where is the camera is on the robot?
    - robot_base: The wheeled part that moves
    - mounting_point: Where the camera is mounted
    - camera_center:

      Based on the lens of the camera. Where pictures are taken from.



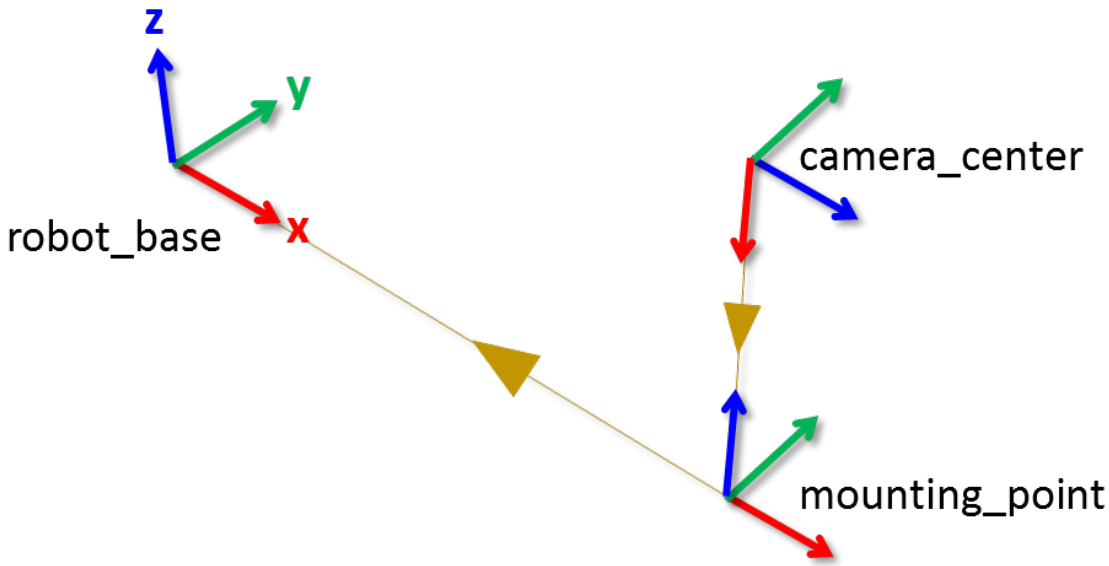
Transformations

- We think of coordinate frames as being relative to each other.
- This relationship is expressed through transformations.
- For our purposes there will be two:
  - Rotation
  - Translation
- If you get more advanced, you will discuss
  - Projections
  - Other advanced transforms!



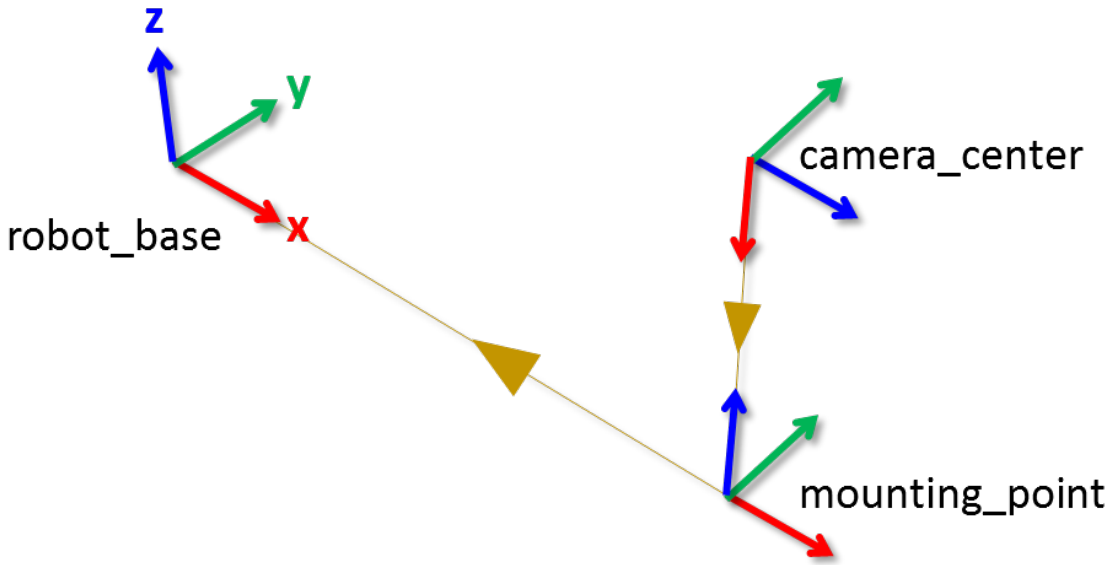
Transformations

- In this image, we can see how rotation and translation relate these coordinate frames.
- Each has an origin at (0,0,0)
- Each represents its rotation such that the vectors face
  - <1,0,0>
  - <0,1,0>
  - <0,0,1>



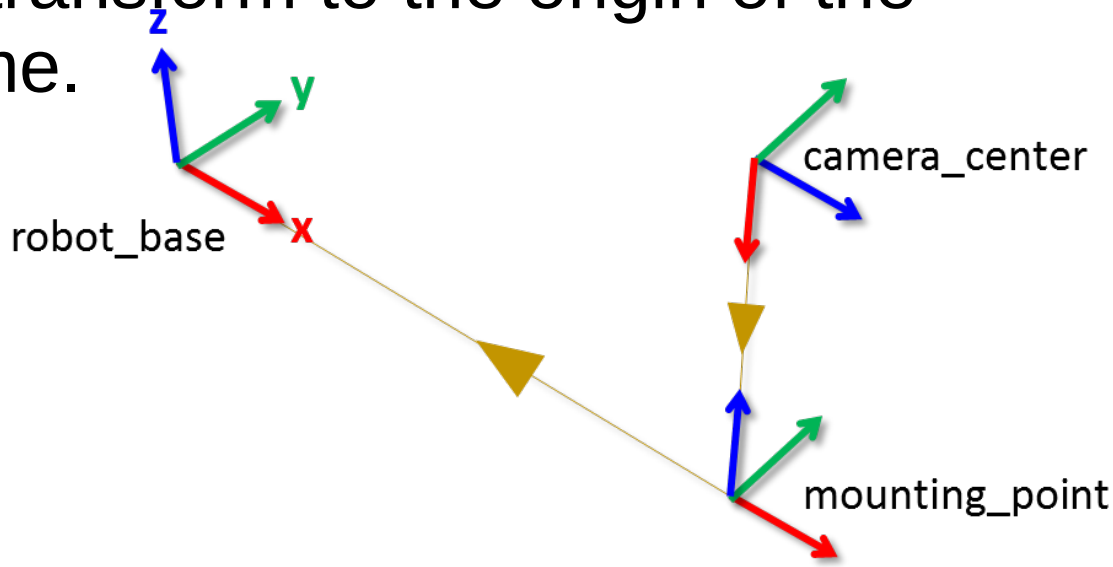
Transformations

- But relative to OTHER coordinate frames, this is not true!!
  - Expressed relative to robot_base
  - camera_center is
    - NOT at (0,0,0)
    - NOT oriented such that:
      - X: <1,0,0>
      - Y: <0,1,0>
      - Z: <0,0,1>
- The relationship between the two is a transformation



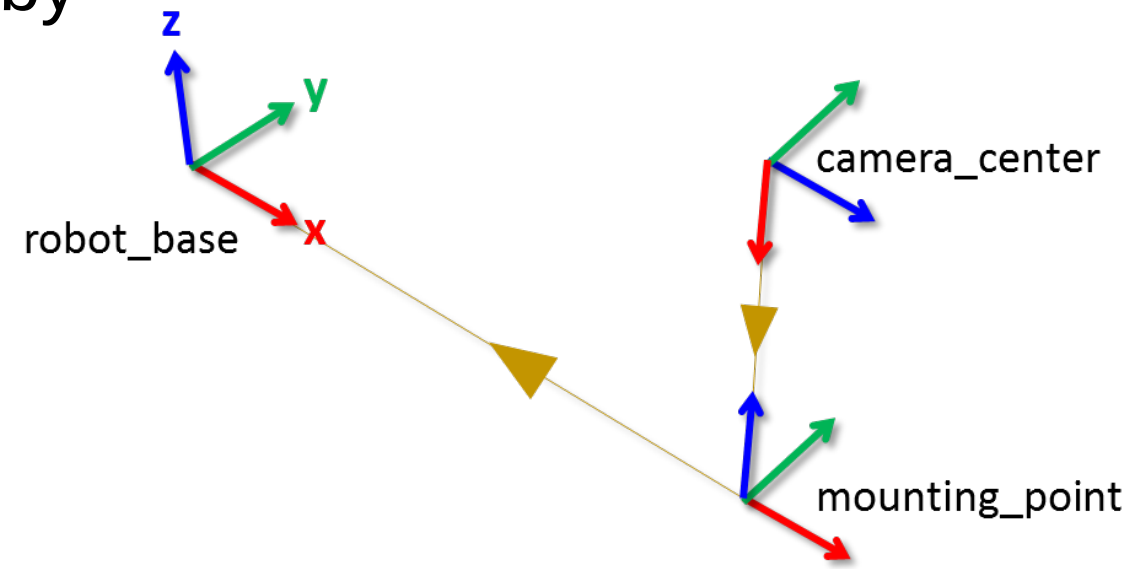
Transformations

- Rotating and translating points expressed relative to robot_base will make them relative to camera_center
- Often we think of important positions on the robot as coordinate frames
  - In this case, we just transform to the origin of the other coordinate frame.



Transformations

- Those transformations are rotation and translation
- In robotics, they are generally combined into a “rigid transformation” which does both at the same time.
  - This will be handled by ROS, so try not to worry about it



Another illustrative example

- Think of a robot arm moving
- Each joint rotates
- Each joint has an offset from the other joints
- Each joint sits in its own coordinate frame
- So tracking the coordinate frames tells you where each joint is, and where the “end effector”, the robot’s hand, is.